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PROBABILISTIC ANALYSIS OF VOLTAGE DEVIATIONS IN THE DISTRIBUTION NETWORK WITH SMALL SYNCHRONOUS GENERATORS

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Abstract - **Fulfilling voltage quality requirements may produce the bottleneck of sending the whole local generation to the medium voltage network. Voltage regulation at the transformer on-load tap changer and small synchronous generators can be simultaneously used for voltage control in the distribution network. To achieve proper results the randomness of active and reactive node loads should be taken into consideration. Solving such problems is the goal of paper. Mathematical background and results of simulations are presented.**

1. INTRODUCTION

Fulfilling voltage quality requirements produces the bottleneck of sending the whole local generation to the medium voltage (MV) network. The voltage regulation at small power station and the HV/MV transformer on-load tap changer can be simultaneously used for voltage control [1-7]. To achieve proper results the randomness of active and reactive node loads should be taken into consideration. The rectangular probability distribution of node loads may be assumed as a pessimistic one. So the node loads are described by Ey - the expected value and My - the covariance matrix. To find the value of node voltages flow equations can be linearized. Rectangular components of node voltages may be treated as the linear transformation of uniformly distributed node loads. All non-linear function of rectangular components of node voltages can be linearized as well. It means that the node voltage magnitudes are the random variables subjected to the normal probability distribution [7]. All considerations are illustrated with the example 20 kV power network with 2.5 MVA synchronous generator.

2. MATHEMATICAL BACKGROUND

2.1. Probabilistic load flow equations

Active and reactive power at all nodes may be treated as random variables. In the most pessimistic case the random values of powers may be treated as rectangular distributed in the range between their min and max

$$P_{min} \le P \le P_{max} \tag{1}$$

$$Q_{min} \le Q \le Q_{max} \tag{2}$$

Random node powers can be characterised by expected values, variances and covariances as follows

- expected value of active power

$$E(P) = (P_{max} - P_{min})/2 \tag{3}$$

- expected value of reactive power

$$E(Q) =(Q_{max} - Q_{min})/2 \tag{4}$$

- variance of active power

$$m_{PP} = \sigma_P^2 = (P_{max} - P_{min})^2/12 \tag{5}$$

- variance of reactive power

$$m_{QQ} = \sigma_Q^2 = (Q_{max} - Q_{min})^2/12 \tag{6}$$

The active and reactive power at a given node may be treated as the independent random variables what means that their covariance is equal to zero

$$m_{PQ} = 0 \quad (7)$$

The variances and covariances create the covariance matrix. The covariance matrix in this case is the diagonal matrix

$$\mathbf{M}_y = \begin{bmatrix} \mathbf{m}_{PP} & 0 \\ 0 & \mathbf{m}_{QQ} \end{bmatrix} \quad (8)$$

The randomness of load powers involves the randomness of the node voltage magnitudes and angles. The easiest way to calculate the expected values, variances and covariances of node voltages is a linearization of load flow equations [7].

The node powers are treated as the random variables, so generally we have

$$\mathbf{y = g(x)} \quad (9)$$

where

$\mathbf{y} = \begin{bmatrix} \mathbf{P} \\ \mathbf{Q} \end{bmatrix}$ – the vector of random node powers,

P – the vector of node active powers,

Q – the vector of node reactive powers,

$\mathbf{x} = \begin{bmatrix} \mathbf{e} \\ \mathbf{f} \end{bmatrix}$ – the vector of random node voltages,

e – the vector of real part of node voltages,

f – the vector of image part of node voltages,

g – the nonlinear function.

The function **g(x)** is a square function and can be linearized. Then we have

$$\mathbf{y - g(Ex) = A(x-Ex)} \quad (10)$$

or

$$\mathbf{\Delta y = A\ \Delta x} \quad (11)$$

where

$\mathbf{\Delta y = y - Ey}$ – the vector of node load deviations,

$\mathbf{\Delta x = x - Ex}$ – the vector of node voltage deviations.

For the expected values we have the following relation

$$\mathbf{Ey = g(Ex)} \quad (12)$$

The unknown expected value of node voltages can be calculated iteratically using the Newton method in the same way as in the case of deterministic load flow

$$\mathbf{Ex_{new} = Ex_{old} + A_{old}^{-1} -[Ey - g(Ex_{old})]} \quad (13)$$

The deviations of node voltages can be calculated using the inverse of the Jacobian at the point of **Ex**

$$\mathbf{\Delta x = A^{-1}\ \Delta y} \quad (14)$$

Then the covariance matrix of the node voltage is equal

$$\mathbf{M_x = A^{-1}\ M_y\ A^{-1\,T}} \quad (15)$$

So, the node voltages are treated as the linear transformation of uniformly distributed node powers. It means that the rectangular components of node voltages are the random variables, which are subjected to the normal probability distribution

$$N(Ex,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(x-Ex)^2}{2\sigma^2}\right] \quad (16)$$

where x is the real or image part of node voltage.

The voltage magnitude is the nonlinear function of real and image part

$$U = \sqrt{e^2 + f^2} \quad (17)$$

After linearization around the point of expected values $[e_0, f_0]$ we obtain

$$\Delta U = \frac{e_0}{U_0}\Delta e + \frac{f_0}{U_0}\Delta f \quad (18)$$

or

$$\mathbf{\Delta U} = \begin{bmatrix} \mathbf{diag(e_0/U_0)} & \mathbf{0} \\ \mathbf{0} & \mathbf{diag(f_0)} \end{bmatrix} \begin{bmatrix} \mathbf{\Delta e} \\ \mathbf{\Delta f} \end{bmatrix} = \mathbf{K\ \Delta x} \quad (19)$$

where

K – the transformation matrix of rectangular components of node voltage deviations into node voltage magnitude deviations.

Then the covariance matrix of the node voltage magnitudes is equal

$$\mathbf{M_U = K\ M_x\ K^T} \quad (20)$$

Knowing the standard deviation we can calculate the probability of constraint violation

$$p_e = p\{z_{min} \le z \le z_{max}\} = F(\tau_b) - F(\tau_a) \quad (21)$$

where F means Laplace integral

$$F(\tau) = \frac{2}{\sqrt{2\pi}} \int_0^\tau \exp(-\tau^2 / 2) dz \qquad (422)$$

and τ is the standardized value of random variable z

$$\tau = (z - Ez)/\sigma \qquad (23)$$

2.3. Probabilistic modelling of tap changing

Regulating transformers play very important role in load flow analysis. One should note that change of transformer ratio affects the node conductances and susceptances in load flow equations.
Transformer ratio changes its value according to the following formula

$$t = t_{min} + (n-1)\, dt \qquad (24)$$

where
- t_{min} - minimal value of transformer ratio,
- n - the tap number,
- dt - discrete step of tap change.

The value of the magnitude of secondary voltage is set to the demanded value U_{kset}. The actual value of U_k is compared with the set value

$$dU = U_k - U_{kset} \qquad (25)$$

The difference dU is checked to be inside dead band ε and the ouput signal e is created

$$e = 0 \quad \text{if} \quad -\varepsilon/2 \le dU \le \varepsilon/2 \qquad (26)$$

$$e = +1 \text{if} \quad dU > \varepsilon/2 \qquad (27)$$

$$e = -1 \text{ if} \quad -\varepsilon/2 > dU \qquad (28)$$

The new value of position of tap changer is calculated according to the following formula

$$n = n_{old} + e \qquad (29)$$

The probability of dead band violation is calculated

$$p_e = p\{-\varepsilon/2 \le dU \le \varepsilon/2\} \qquad (30)$$

If $p_e > 0.95$ then e =0. Otherwise the probabilities of left and right side violation are calculated

$$p_{eleft} = p\{-\infty < dU < -\varepsilon/2\} \qquad (31)$$

$$p_{eright} = p\{\varepsilon/2 < dU < +\infty\} \qquad (32)$$

If $p_{eleft} > 0.95$ then e = -1. If $p_{eright} > 0.95$ then e = +1. Otherwise e=0. New value of discrete tap is calculated from the deterministic formula

$$n = n_{old} + e \qquad (33)$$

The probability of the violation of the lower limit equals

$$p_{Umin} = p\{-\infty < U_0 + \Delta U < U_{min}\} \qquad (34)$$

and the probability of the violation of the upper limit is

$$p_{Umax} = p\{U_{max} < U_0 + \Delta U < +\infty\} \qquad (35)$$

3. EXAMPLE CALCULATIONS

To illustrate all considerations an example power system shown in Figure 1 has been analysed. It known that the customer bus voltage must not violate +10%, -10% of the nominal voltage U_N. A small generator can work with the constant power factor. To find voltage level the analysis of probabilistic load flow may be made instead of the deterministic one.
The analysed example power system is shown in Fig. 1. The lines data are given in Table 1.

TABLE 1. Lines data

From	To	R Ω	X Ω	B μS
GPZ-20kV	o223	0.069	0.047	39.480
o223	1425	0.318	0.264	2.166
1425	R961-28	0.304	0.148	0.920
1426	slup21	0.364	0.303	2.484
slup21	R961-11	0.088	0.073	0.600
slup21	1434o610	0.202	0.168	1.380
1434o610	R961-27	0.602	0.293	1.820
1434o610	slup31	0.396	0.329	2.700
slup31	R961-17	0.096	0.047	0.291
slup31	GEN-20kV	0.033	0.012	7.400
slup31	1446o692	0.572	0.476	3.900
1446o692	R961-13	0.048	0.017	0.104
1434o692	slup45	0.251	0.209	1.710
slup45	R961-14	0.662	0.232	1.435
slup45	R961-1	1.031	0.858	7.029
R961-1	R961-2	0.685	0.570	4.671
R961-2	1458o693	0.374	0.311	2.550
1458o693	R961-15	0.572	0.278	1.729

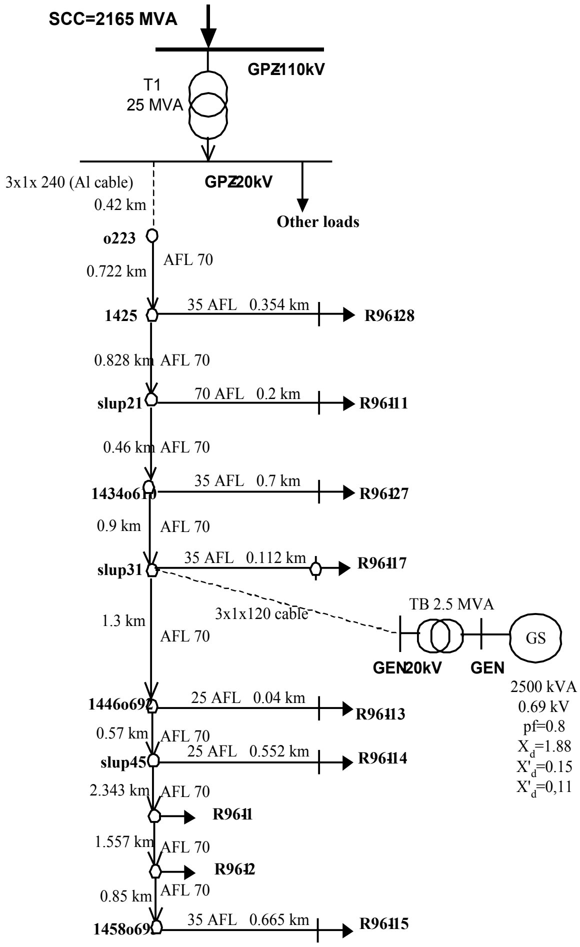

Figure 1. The scheme of power system

The transformers data of the power system are given in Table 1.

TABLE 2. Transformers data

Variable	Unit	T-1	TB
Sn	MVA	25	2.5
UnH	kV	115+/-16%	20+/-5%
UnL	kV	22	0.69
Pcu	kW	130	20
uk	%	11	6
Pfe	kW	24	5
Io	%	0.38	1

The relevant load and generation of the analysed MV power system are given in Table 3. A small generator can work with constant power factor. From the point of efficieny the nominal power factor should be chosen. From the voltage requirements in the network the power factor equal 1 is always chosen. It was assumed that active and reactive load at nodes can change +/-50% due to random demand. So the node voltages should be treated as random variables.

TABLE 3. Node load and generation

Node	Pgen MW	Qgen MVAr	Pload MW	Qload MVAr
GPZ-20kV			7.44	1.29
o223				
1425				
R961-28			1.5	0.6
slup21				
R961-11				
1434o610				
R961-27			1.8	0.8
slup31				
R961-17			1.5	0.6
GEN-20kV			0.2	0.08
GEN	2.0	1.5 or 0		
1446o692				
R961-13			0.5	0.2
slup45				
R961-14			0.1	0.04
R961-1			0.2	0.08
R961-2			0.1	0.04
1458o693				
R961-15			0.15	0.06

Voltage regulation at GPZ-20kV bus is achieved using on-load tap-changers having 25 taps giving a range of +/-16% with tap 13 corresponding to the nominal ratio of 115/22 kV. The dead band is equal 2.0%. The maximal, minimal and step tap ratio are given in Table 4.

TABLE 4. Tap-changer data

Nominal tap ratio	Max. tap ratio	Min. tap ratio	Step tap ratio	Dead band
0.9504	1.1025	0.7983	0.0127	0.02

The results of probabilistic flow analysis with the following generation was done

- Pgen = 0 MW, Qgen = 0 MVAr - no local generation,
- Pgen = 2.0 MW, Qgen = 1.5 MVAr - the local generation with the nominal power factor pf = 0.8,
- Pgen = 2.0 MW, Qgen = 0 MVAr - the local generation with power factor pf = 1.

All the per unit (p.u.) values are related to the base power of 100 MVA and the node nominal voltage.
The results of probabilistic load flow without local generation are shown i Table 5 and Figure 2.

TABLE 5. Probabilistic load flow with no generation

Node	GPZ-20kV	GEN-20kV	GEN	R961 -15
EU	1.0927	1.0739	1.0736	1.0692

Node	GPZ-20kV	GEN-20kV	GEN	R961-15
sigmaU	0.0028	0.0041	0.0041	0.0042
Umax = U+3sigmaU	1.1011	1.0862	1.0859	1.0818
Umin = U-3sigmaU	1.0843	1.0616	1.0613	1.0566
probability of (0.9<U<1.1)	0.9954	1.0000	1.0000	1.0000

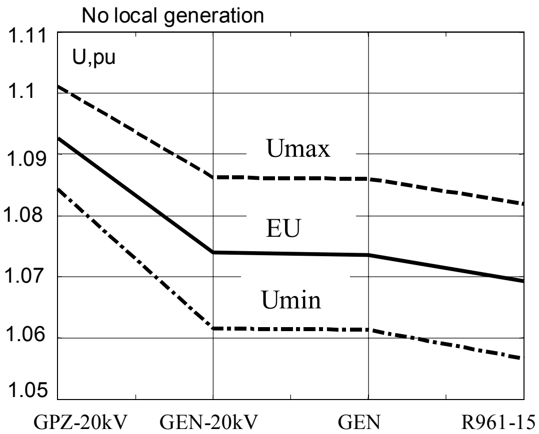

Figure 2. Random node voltage magnitudes without local generation

The generator can work with the nominal power pf = 0.8. The results of probabilistic load flow in such a case are given in Table 6 and Figure 3.

TABLE 6. Probabilistic load flow with local generation and pf = 0.8.

Node	GPZ-20kV	GEN-20kV	GEN	R961-15
EU	1.1	1.0921	1.1235	1.0873
sigmaU	0.0027	0.0039	0.0038	0.0040
Umax = U+3sigmaU	1.1081	1.1038	1.1349	1.0993
Umin = U-3sigmaU	1.0919	1.0804	1.1121	1.0753
probability of (0.9<U<1.1)	0.5000	0.9786	0	0.9993

A dilemma can arise between the required active power output of the local generation and the need to maintain system voltage within –10%/+10% limits. For the above reasons the powr factor equal 1 is chosen. The results of probabilistic load flow associated with the power factor pf = 1 are given in Table 7 and Figure 4.

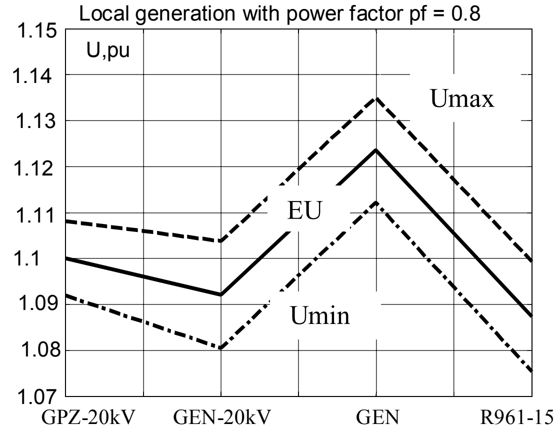

Figure 3. Random node voltage magnitudes with Pgen = 2.0 MW and pf = 0.8

TABLE 7. Probabilistic load flow with local generation and pf = 1.

Node	GPZ-20kV	GEN-20kV	GEN	R961-15
EU	1.0935	1.0810	1.0803	1.0762
sigmaU	0.0028	0.0040	0.0040	0.0041
Umax = U+3sigmaU	1.1019	1.0930	1.0923	1.0885
Umin = U-3sigmaU	1.1019	1.0930	1.0923	1.0885
probability of (0.9<U<1.1)	0.9899	1.0000	1.0000	1.0000

From the presented results of probabilistic load flows in the example power system it is clearly seen that the local generation increases the node voltage in the network. The lowest increase of node voltage can be achieved when the local generator works with the power factor equal one.

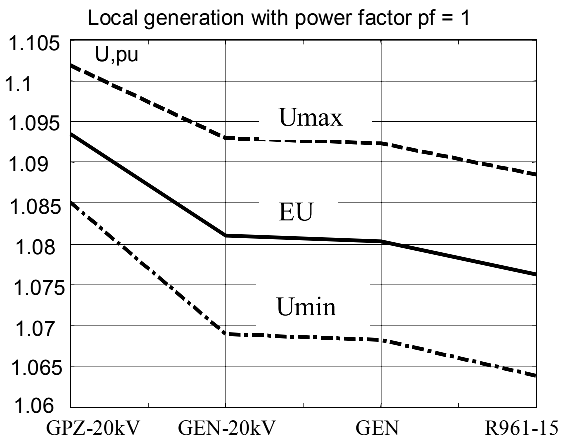

Figure 4. Random node voltage magnitudes with local generation and pf = 1.

4. CONCLUSIONS

1. Local generation connected to the MV network is a new challenge for power planners and operators. The correct strategy for managing voltage regulation in the presence of local generation depends on the operation mode of the local generator and the on-load tap-changers control.
2. New requirements and constraints in voltage regulation in the vicinity of dispersed generation imply using new methods of power network analysis. Especially, the uncertainty of load demand may be taken into account.
3. In this paper, the probabilistic method of load flow is proposed for steady state analysis of local generation connected to the MV network. The results obtained from the probabilistic load flow study give much more information than obtained from the deterministic load flow study.

5. REFERENCES

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